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DOUBLE SCATTERING CORRECTIONS FOR THE THEORY OF THE SUN'S AUREOLE

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16. ABSTRACT

Double scattering corrections for aureole radiances are calculated by adding the effects of two successive single scatterings. Atmospheric absorption, polarization, and variation of refractive index with altitude are ignored. Corrections due to spherical atmosphere have been taken into account by the use of a generalized Chapman function. Realistic scattering phase functions based upon the Lorenz-Mie theory and model altitude-size distribution are used. The model distribution is assumed to be representable in terms of two separable particulate components. We find that for a moderately clear day, $\tau(0) \cong 0.5$, and for forward scattering angles, the radiance, B_2, due to double scattering is less than 6 percent of that due to single scattering.

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DOUBLE SCATTERING CORRECTIONS FOR THE THEORY OF THE SUN'S AUREOLE

I. INTRODUCTION

In our earlier work [1,2] we demonstrated a technique by which we could obtain the size distribution (SD), n(r), and the vertical profile, $\rho(y)$, of the atmospheric aerosols from the sky radiance measurements in the region of the sun's aureole. The theory was based on the assumption that only single scattering (SS) is present, which would be the case when the sky is clear. However, when the atmosphere in the line of sight of the sun is somewhat hazy, the SS approximation may not be valid in general, and higher orders of scattering may have to be included to explain the results.

The precise conditions at which multiple scattering (MS) corrections are needed are not yet established. Piaskowska-Fesenkova [3] in an extensive series of experimental and theoretical studies has shown that higher order scattering is practically insignificant in their aureole results. E. de Bary [4] in her extensive theoretical studies has also shown that the influence of multiple scattering is negligible close to the sun. On the other hand, according to Van de Hulst [5], SS prevails when the optical depth, τ, of the medium is less than 0.1; second order scattering or double scattering (DS) should be taken into account when $0.1 < \tau < 0.3$; and all orders of MS for $\tau > 0.3$. Whether these conditions should apply to aureole calculations is not clear. To resolve this possible contradiction, we have carried out an explicit calculation of the DS contributions to the radiance, B_2, as compared to the SS contribution, B_1. The general approach we take is a further development of the work of Dowling and Green [6] and Deepak [7,8], which are rather straightforward but tedious extensions of the first order scattering calculations.

In the present case we treat the case when the wavelength and atmospheric conditions are such that DS is present as a correction. We use the simplifications of the SS theory to approximately determine n(r) and $\rho(y)$ from sky radiance data. Since there are certain angular regions in the sky where the DS is negligibly small compared to the SS, we make use of this fact in our solution of the problem as explained below.

The steps for attacking the DS problem are the following:

1. First, we should get some idea of the angular distance θ°_{SS} from the sun in which DS $\ll$ SS. This can be done through preliminary computer studies by using known models and test cases of n(r) and $\rho(y)$.

2. Next, we interpret the radiance data along the almucantar in this region by SS alone (as described in References 1 and 2) and obtain approximations for n(r) and $\rho(y)$.

3. Finally, we substitute these functions back into the expressions for the total radiance, B_{1+2}, due to SS and DS, and readjust the parameters in the expressions for $n(r)$ and $\rho(y)$ to best fit the data.

A brief description of the theory is given in the next section.

II. THEORETICAL CONSIDERATIONS

The approach adopted for the DS and the concepts and assumptions used in obtaining the expressions for the sky radiance due to the various orders of scattering have been explained in detail in References 7 and 8. The basis of our theoretical treatment is (1) to treat the radiance resulting from MS as the sum of contributions due to the various orders of scattering, and (2) to consider the Nth order scattering as the result of N single scatterings.

To obtain a clearer physical insight into the problem, we invoke the following reasonable simplifying assumptions:

1. The atmosphere is composed of air molecules and (various species of) aerosols.

2. The atmosphere is vertically homogeneous; i.e., the altitude-size distribution is a separable function of the form

$$\eta(r,y) = n(r)\,\rho(y),\ [\mathrm{cm}^{-3}\ \mu^{-1}] \quad , \tag{1}$$

which represents the case when the size distribution, $n(r)$, of the aerosols does not vary with altitude. $\rho(y)$ is a dimensionless parameter representing the altitude dependence of the total concentration (density) of the particles.

3. The atmosphere is horizontally homogeneous, which means that in any horizontal plane at an altitude y, the atmospheric composition remains constant. The composition may change from plane to plane which would be the case of a vertically inhomogeneous atmosphere. Horizontal homogeneity need not imply vertical homogeneity or vice versa.

4. The effects of absorption are ignored by choosing to work in the spectral regions for which the atmospheric absorption is negligibly small, and only scattering predominates.

5. The effects of polarization are ignored. This would be a perfectly reasonable assumption for SS since we confine our sky radiance measurements to the solar aureole region so that it is essentially the case of forward scattering. However, since DS is also present, practically all angles of scattering have to be considered.

6. To obtain the geometrical relations involved, the atmosphere is treated as plane parallel, and the effect of the spherical earth, which becomes important whenever the scattering angle involved is greater than 75 degrees, is taken into account by the use of the concept of generalized Chapman functions [6,9].

7. The effects of ground reflection are also ignored in this treatment.

The effects of absorption and polarization may be introduced into the problem by the respective concepts of volume absorption coefficient and the Stokes' polarization parameters.

III. SKY RADIANCE DUE TO DS

The geometry of the problem is illustrated in Figure 1. The volume element, dV_1, which in the case of SS is illuminated by direct sunlight, is now illuminated by light scattered by another volume, dV_2, anywhere else in the surrounding space. The dihedral angles between the detector-zenith and zenith dV_2 planes is ω_1, and that between dV_1-zenith and zenith-sun planes is ω_2. These are related to the dihedral angle ω_1 between the detector-zenith and the zenith-sun planes by the relation

$$\omega_2 = \pi + \omega_1 - \omega_1' \quad . \tag{2}$$

The first scattering takes place at dV_2 and the second at dV_1 (Fig. 1):

$$dV_2 = R_2^2 \sin\phi_2 \, d\phi_2 \, d\omega_1' \, dy_2 \sec\phi_2 \tag{3a}$$

and

$$dV_1 = R_1^2 dR_1 d\Omega = R_1^2 \sin\phi_1 \, d\phi_1 \, d\omega_1 \, dy_1 \sec\phi_1 \quad . \tag{3b}$$

The scattering angles ψ_2 and ψ_1, at the dV_2 and dV_1 respectively, are related to the directional angles as given below:

$$\cos\psi_2 = \cos\theta_1 \cos\phi_2 + \sin\theta_1 \sin\phi_2 \cos(\omega_1 - \omega_1') \tag{4a}$$

and

$$\cos\psi_1 = \cos\phi_1 \cos\phi_2 - \sin\phi_1 \sin\phi_2 \cos\omega' \quad . \tag{4b}$$

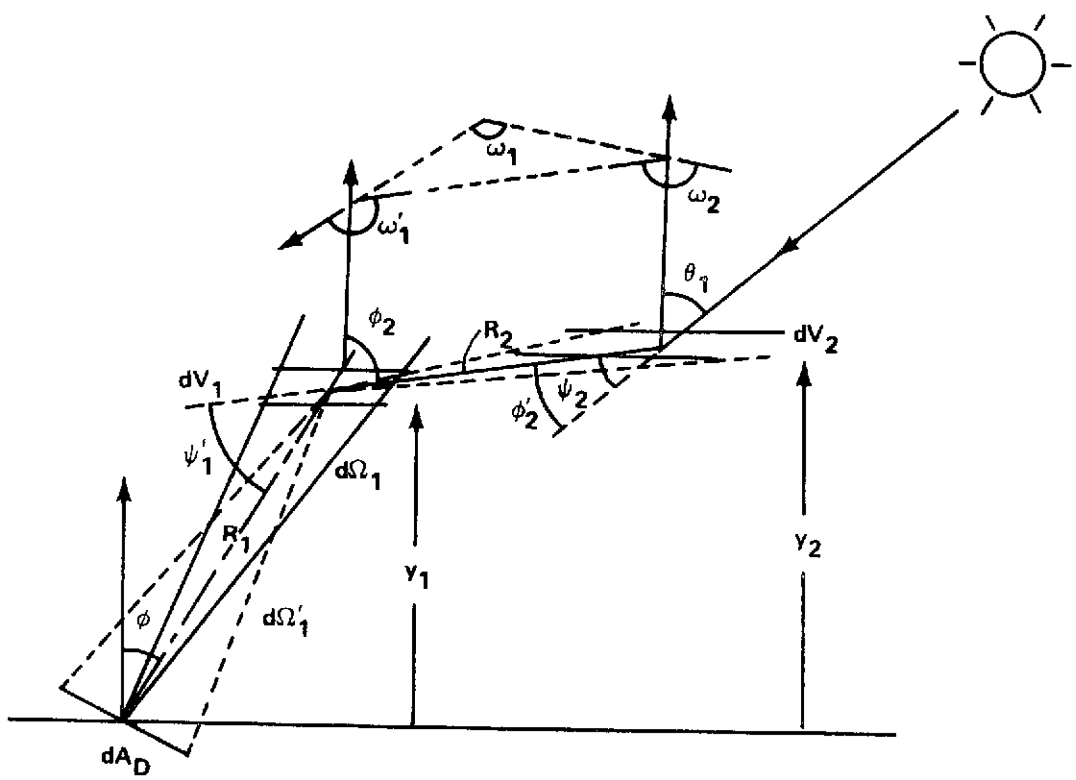


Figure 1. Geometry illustrating the double scattering of sunlight by the atmosphere.

The altitude dependent volume scattering functions are

$$F'(\psi_2, y_2) = \sum_i F_i(\psi_2)\,\rho_i(y_2) \quad , \quad [\mathrm{km}^{-1}\ \mathrm{sr}^{-1}] \tag{5a}$$

and

$$F'(\psi_1', y_1) = \sum_j F_j(\psi_1')\,\rho_j(y_1) \quad , \quad [\mathrm{km}^{-1}\ \mathrm{sr}^{-1}] \quad , \tag{5b}$$

where i and j represent the species of scatterers that significantly contribute to the sky radiance.

The optical depths involved are determined for each path just as in the SS analysis. By tedious but straightforward mathematical manipulation, we may express the total sky radiance due to DS in the form

$$B_2(\lambda,\phi_1,\omega,\theta_1) = G(\phi_1,\lambda) \int_0^\infty e^{-\Sigma_i \tau_i(y_1) D_{1i}} F'^{(2)}(\psi_1,y_1)\, dy_1 \quad , \qquad (6)$$

where

$$D_{1i} = - S_i(\phi_1) + S_i(\theta_1), \; G = H_0(\lambda)[e^{-\Sigma_i \tau_i(0) S_i(\phi_1)}] \sec\phi_1 \quad ,$$

$$F'^{(2)}(\psi_1,y_1) = \int_0^\pi \int_0^{2\pi} \int_0^\infty F'(\psi_2,y_2)\, F'(\psi_1',y_1)\, e^{\Sigma_i[\tau_i(y_1) - \tau_i(y_2)] D_{2i}}$$

$$\times \sec\phi_2 \sin\phi_2 \, d\phi_2 \, d\omega' \, dy_2 \quad , \qquad (7)$$

and

$$D_{2i} = S_i(\phi_2') - S_i(\theta_1) \quad .$$

S_i represents secant functions for angles $\leqslant$ 70 degrees, and generalized Chapman functions (defined later) for angles greater than 70 degrees. $F'^{(2)}$ may be referred to as the "equivalent double scattering function" (EDSF). This form, equation (6), is similar to the corresponding result for SS, except that the volume scattering function for SS is replaced by $F'^{(2)}(\psi_1,y_1)$, the scattering function for a unit volume of the atmosphere at altitude y_1 due to DS, which is defined below. To simplify the discussion, we limit the number of scattering species to air molecules and two aerosol components denoted by p and q. Thus, in equation (7) we let

$$F'(\psi_2,y_2) = k_a(\lambda)P_a(\psi_2)P_a(y_2) + k_p(\lambda)P_p(\psi_2)\,\rho_p(y_2)$$

$$+ k_q(\lambda)P_q(\psi_2)\,\rho_q(y_2) \qquad (8)$$

and use a similar expression for $F'(\psi_1',y_1)$.

The total sky radiance due to SS and DS now may be written

$$B_{1+2}(\lambda,\phi_1,\omega_1,\theta_1) = B_1 + B_2 = G(\phi_1,\lambda) \int_0^\infty e^{-\Sigma_i \tau_i(y_1) D_{1i}}$$

$$\times \; [F'(\psi_1,y_1) + F'^{(2)}(\psi_1,y_1)] \, dy_1 \quad . \tag{9}$$

This shows that the presence of DS results in modifying $F'(\psi_1,y_1)$ to

$$[F'(\psi_1,y_1) + F'^{(2)}(\psi_1,y_1)] \quad . \tag{10}$$

Introducing the integrated thickness functions

$$F_a(D_{2i}) = \frac{1}{w_a(0)} \int_0^\infty e^{-\Sigma_i \tau_i(y_2) D_{2i}} \rho_a(y_2) dy_2 \quad , \tag{11a}$$

$$F_p(D_{2i}) = \frac{1}{w_p(0)} \int_0^\infty e^{-\Sigma_i \tau_i(y_2) D_{2i}} \rho_p(y_2) dy_2 \quad , \tag{11b}$$

and

$$F_q(D_{2i}) = \frac{1}{w_q(0)} \int_0^\infty e^{-\Sigma_i \tau_i(y_2) D_{2i}} \rho_q(y_2) dy_2 \tag{11c}$$

where

$$w(y) = \int_y^\infty \rho(y)\, dy \quad ,$$

and again using tedious but straightforward algebra, we may express B_2 in the form

$$B_2 = \Sigma B_{ij} \quad , \tag{12}$$

where $i = 1, 2$, or 3 denotes a, p, or q, and where each B_{ij} has the form

$$B_{ij} = \int_0^{\pi} \int_0^{2\pi} \sin\phi_2 \, d\phi_2 \, d\omega', \ \sec\phi_2' \ J_{ij} \quad , \tag{13}$$

where

$$J_{ij} = \tau_i(0)\tau_j(0)P_i(\psi_1')P_j(\psi_2)F_i(D_{2i}) \ F(D_{21j}) \tag{14}$$

and

$$D_{21j} = D_{1j} - D_{2j} \quad .$$

Here each term yields a DS contribution due to scattering among the various species. Let us consider now an almucantar scan ($\phi_1 = \theta_1$), then $D_{1i} = 0$, $D_{2i} = D_{21i}$, hence $F_a(0) = F_p(0) = F_q(0) = 1$ and equations (12) and (13) simplify accordingly.

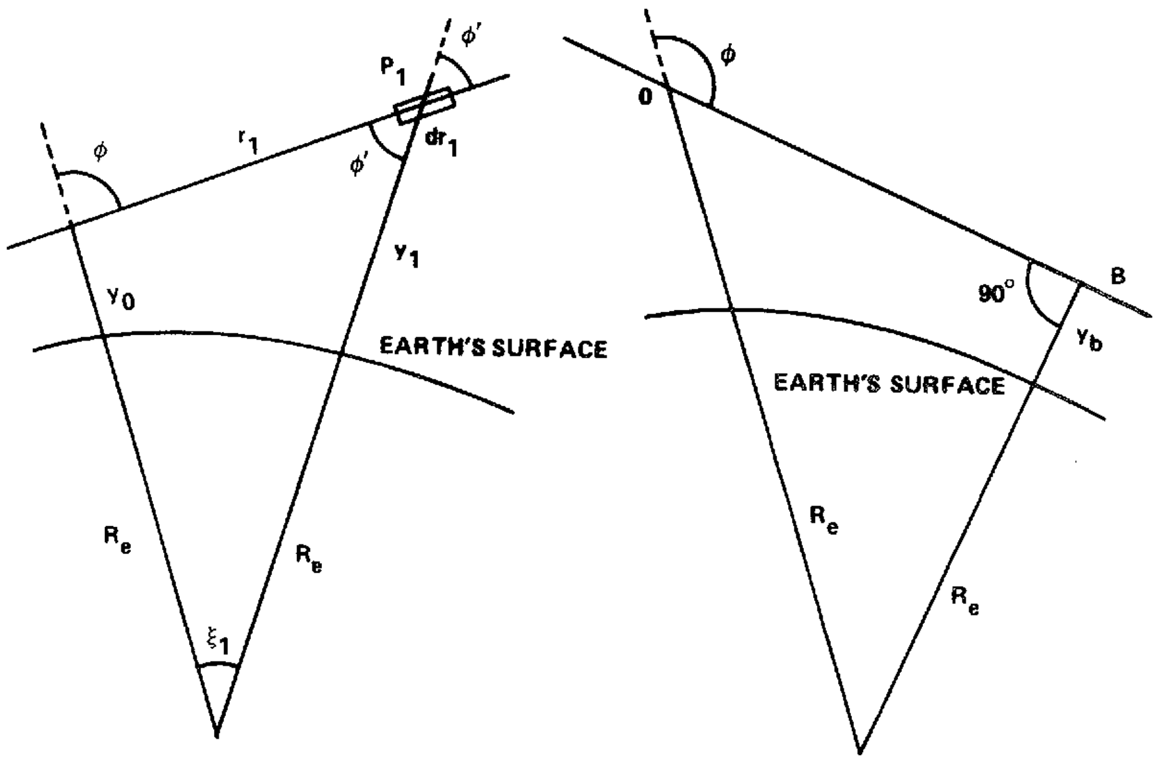
Notice that when $\phi_1 = \theta_1$, the y-dependence vanishes from the expression for the SS radiance, B_1, but not from that for the DS radiance.

IV. SPHERICAL EARTH CORRECTIONS

In the computation for B_2, one encounters the singular secant function since integration over elevation angles, ϕ, from 0 to π is involved. This integrable singularity is taken care of by replacing the secants by the generalized Chapman function (GCF) (described later in the Appendix and in References 6 and 9) for angles, ϕ, close to or greater than $\pi/2$. The GCF is also referred to as the "air mass" in astronomy. It is essentially an average of the secant function taken over the size distribution of each of the scattering species and is defined by

$$S(\phi, y_0) = \frac{\int_{y_0}^{\infty} \rho(y_1) \, dy_1 \ \sec\phi'}{\int_{y_0}^{\infty} \rho(y_1) \, dy_1} \quad , \tag{15}$$

where ϕ is the local zenith angle of the line of sight at volume element P_1 (Fig. 2) and $\rho(y_1)$ is the density altitude distribution. We have used the detailed approach roughly following that of References 6 and 9, which considers three regions, $\phi < 70$ degrees < 85 degrees < 90 degrees and a fourth region beyond 90 degrees.



(a) $\phi \leqslant 90$ degrees (b) $\phi > 90$ degrees

Figure 2. Geometry of the generalized Chapman functions.

V. COMPUTATIONAL SCHEME AND RESULTS

The computations for B_2 involve four integrations $\phi(0 \text{ to } \pi)$, $\omega(0 \text{ to } 2\pi)$, $y_1(0 \text{ to } \infty)$ and $r_1(0 \text{ to } \infty)$. A two-dimensional integration over ϕ and ω can be performed very economically and accurately by Conroy's method [10]. Integration over y_1 and r_1 is performed by the use of the 12-point Laquerre integration scheme. Computations were performed using GCF formulae to take care of the singularity of the $\sec\phi'$ for $\phi_1' = 90$ degrees.

We use the oversize distribution function of the form

$$N(r) = N_0 \left[1 + (r/a)^{\nu}\right]^{-1} \quad , \tag{16}$$

where a is a size parameter, ν is a dispersion parameter, and N_0 is the magnitude parameter. The size distribution is then given by $n(r) = - dN/dr$.

The altitude distrbutions are characterized by a form which contains a flat component and a peaked component described by

$$\rho_j(y) = \frac{\beta_1}{1 + e^{(y - y_1)/h_1}} + \beta_2 \frac{4e^{(y - y_2)/h_2}}{\left[1 + e^{(y - y_2)/h_2}\right]^2} , \tag{17}$$

where β_1 and β_2 control the relative importance of the two components, y_1 is the half-fall-off point of the first term, y_2 is the peak position of the second term, and h_1 and h_2 are diffuseness parameters.

The values of B_1 and B_2 in the almucantar and sun vertical at various ψ for the August 18th event are given in Table 1. One notices that for the clear day of August 18th, 1971, the value of $B_2/B_1 \leqslant 2$ percent for ψ angles up to 20 degrees. The results show that in the case of sun vertical scan as one goes toward the horizon ($\phi_1 > \theta_1$) the values of B_1 and B_2 seem to come closer together; i.e., B_2/B_1 increases, which is the correct trend. The ratio B_2/B_1 shows an increase for angles up to 25 degrees as we go away from the sun. Our calculations for a less clear day of May, 1970 when $\tau(0) = 0.6$ indicate the DS contribution to the sky radiance in the solar aureole is within 4 percent of the SS radiance up to angles of 20 degrees.

Thus, our conclusion tends to support the results of E. de Bary [4] and Piaskowska-Fesenkova [3] that multiple scattering is insignificant in aureole studies under reasonably clear sky conditions.

TABLE 1. SS AND DS RADIANCE (THEORETICAL) IN THE ALMUCANTAR AND THE SUN VERTICAL FOR EVENT 1 (AUGUST 18, 1971), GAINESVILLE, FLORIDA

Data: θ_1 = 39.6 degrees, λ = 550 nm, $\tau_a(0)$ = 0.096, $\tau_p(0)$ = 0.182, $\tau(0)$ = 0.278

Almucantar Scan ($\phi_1 = \theta_1$)	Scattering Angle	Sky Radiance (Theoretical) (W/cm² -sr-μm)		
ω_1 (degrees)	ψ_1 (degrees)	B_1	B_2	B_2/B_1
2	1.275	0.667	1.099(-3)	0.165(-2)
6	3.824	0.306	1.085(-3)	0.354(-2)
10	6.384	0.210	1.062(-3)	0.505(-2)
20	12.71	0.124	1.077(-3)	0.867(-2)
Sun Vertical ($\omega_1 = 0$)				
ϕ_1				
19.6	20.0	0.066	0.659(-3)	0.99(-2)
29.6	10.0	0.135	0.826(-3)	0.61(-2)
37.6	2.0	0.518	1.07(-3)	0.207(-2)
41.6	2.0	0.543	1.17(-3)	0.215(-2)
45.6	6.0	0.243	1.41(-3)	0.579(-2)
49.6	10.0	0.173	1.57(-3)	0.910(-2)
59.6	20.0	0.109	2.10(-3)	1.92(-2)

APPENDIX

THE GENERALIZED CHAPMAN FUNCTION

A brief description and a summary of the expressions for the GCF(s) pertaining to the various angular regions are given below.

If the form of $\rho(y_1)$ in equation (15) is exponential, then $S(\phi, y_0)$ would reduce to the usual Chapman function.

From the geometry (Fig. 2), one obtains

$$\sec \phi' = \left[1 - \left(\frac{R_e + y_0}{R_e + y_1}\right)^2 \sec \phi_1\right]^{-\frac{1}{2}} , \qquad \text{(A-1)}$$

which in terms of the variables

$$t = \frac{y_1 - y_0}{h_1} \text{ and } x = \frac{R_e + y_0}{h_1} \qquad \text{(A-2)}$$

becomes

$$\sec \phi' = f(t) = \frac{1 + (t/x)}{[\cos^2 \phi + 2(t/x) + (t/x)^2]^{\frac{1}{2}}} \qquad \text{(A-3)}$$

This expression has a $t^{-\frac{1}{2}}$ singularity near the origin as ϕ_1 goes to 90 degrees. This is an integrable singularity and can be dealt with by using the forms for the GCF [9] that are summarized below.

For $\phi < 70$ degrees, the generalized Chapman function

$$S(\phi) = \sec \phi \quad . \qquad \text{(A-4)}$$

For 70 degrees $< \phi <$ 85 degrees, one uses the following expression:

$$S(\phi, y_0) = S_e(\phi, y_0) + S_r(\phi, y_0) \quad , \tag{A-5}$$

where, if $\beta = x^{1/2} \cos \phi$, the essential part is given by

$$S_e(\phi, y_0) = x^{1/2} \int_0^\infty \frac{\rho(t)\, dt}{(\beta^2 + 2t)^{1/2}} \tag{A-6}$$

and the residual part is given by

$$S_r(\phi, y_0) \cong f_e(\bar{t}) \frac{\bar{t}}{x} \left[1 - \frac{\bar{t}}{2x^{1/2}} f_e(\bar{t}) \right] \quad , \tag{A-7}$$

where

$$f_e(t) = \frac{x^{1/2}}{(\beta^2 + 2t)^{1/2}} \tag{A-8}$$

and

$$\bar{t} = \frac{\int_0^\infty t\, \rho(t)\, dt}{\int_0^\infty \rho(t)\, dt} \quad . \tag{A-9}$$

For 85 degrees $< \phi \leqslant 90$ degrees, one can use

$$S_e(\phi, y_0) \cong x^{1/2} \int_0^\infty \rho(t')\, dZ \quad , \tag{A-10}$$

where

$$t' = {}^1/_2\ [(Z + \beta)^2 - \beta^2] \quad . \tag{A-11}$$

For 90 degrees $< \phi \lesssim 120$ degrees (Fig. 2b)

$$S(\phi, y_0) = 2\left[\frac{w\,(y_b)}{w\,(y_0)}\right] S\left(\frac{\pi}{2}, y_b\right) - S(\pi - \phi, y_0) \quad , \tag{A-12}$$

where

$$y_b = (R_e + y_0) \sin\phi - R_e \tag{A-13}$$

and y_b is then greater than zero.

When $y_b \leqslant 0$, the line-of-sight is tangential to or is cut off by the earth's surface, then one uses the form

$$S(\phi, y_0) = \left[\frac{w(y_1)}{w(y_0)}\right] S(\pi - \phi, y_1) - S(\pi - \phi, y_0) \quad , \tag{A-14}$$

where

$$y_1 = [r_2{}^2 + 2r_2 (R_e + y_0) \cos\phi + (R_e + y_0)^2]^{1/2} - R_e \quad . \tag{A-15}$$

For $\phi > 120$ degrees,

$$S(\phi_0) = \sec\phi \quad .$$

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APPROVAL

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

for Gerd Fingerhood

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